CSCE 52803: Assignment 2

Due 11:59pm Friday, September 25, 2026

<u>NOTE: All graphs are simple graphs by default.</u>

1. What are the decision versions of following problems?

(1) Given a complete weighted graph, find the shortest path that visits each vertex exactly once and returns to the starting point.

(2) A dominating set for a graph $G = (V, E)$ is a subset $D \subseteq V$ such that every vertex not in D is adjacent to at least one vertex in D. Given a graph, find the minimum dominating set.

(3) Graph coloring is to color the vertices of a graph such that no two adjacent vertices are of the same color. Given a graph, find the graph coloring that uses the minimum number of colors.

2. What are the optimization versions of following problems?

(1) Given a knapsack of capacity S, n items of volumes v_i, weights w_i, and an integer K, is there a packing of total weight at least K?

(2) Given a set A of integers, is there a subset $B \subseteq A$ such that $\sum_{i \in B} i = \sum_{i \in A \setminus B} i$?

(3) Given a finite set I of items, a size $s(i) \in \mathbb{Z}^+$ for each $i \in I$, a positive integer bin capacity B, and an integer K, is there a partition of I into at most K disjoint sets $I_1, \dots, I_K$ such that the sum of the sizes of the items in each I_j is B or less?

3. The vertex cover problem is defined as: given a graph G and an integer K, is there a vertex cover of size at most K?

The set cover problem is defined as: given a set X, a family of subsets of X, $\mathcal{F}$, such that $X = \cup_{F \in \mathcal{F}} F$, and an integer K, is there a set cover of size at most K?

Prove that the set cover problem is NP-complete by showing that:

(1) The set cover problem is in NP; and

(2) The vertex cover problem is polynomial-time reducible to the set cover problem, i.e., vertex cover $\leq_p$ set cover.

4. Consider the problem in Question 2(1). Prove that this problem is NP-complete by reducing from the subset sum problem.

The subset sum problem is defined as: given a set A of positive integers and a positive integer Q, is there a subset $X \subseteq A$ such that $\sum_{x \in X} x = Q$?

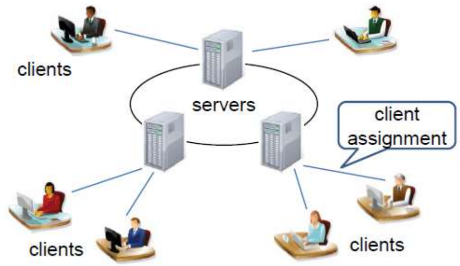

Figure 1: Distributed server architecture.

5. In a distributed server architecture (see Figure 1), clients are assigned to servers and interact with one another through their assigned servers. When Client c_1 wants to communicate with Client c_2, it first sends the message to c_1's server. This server then forwards the message to c_2's server, which finally delivers the message to c_2. The total network latency in this communication process is given by

$$L(c_1, c_2) = d\big(c_1, s(c_1)\big) + d\big(s(c_1), s(c_2)\big) + d(s(c_2), c_2)$$

where $s(\cdot)$ is a client assignment that maps from clients to servers, and $d(\cdot,\cdot)$ is the network latency between the two network devices.

Consider the following optimization problem: Given a weighted graph $G = (V, E)$ where V contains a set of clients C and a set of servers S, and a weight/latency $d(u, v)$ is associated for each edge $(u, v) \in E$, find a client assignment $s: C \mapsto S$ that minimizes the maximum latency between any pair of clients, i.e.,

$$\text{minimize} \max_{c_i, c_j \in C} L(c_i, c_j)$$

(1) What is the decision version of this problem?

(2) Show that the above decision version is in NP.